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# Tidal Dissipation in the Moon

WILLIAM M. KAULA

NASA. Goddard Space Flight Center, Greenbelt, Maryland

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*Abstract.* Dissipation of tidal energy in the moon was calculated under the assumption that it can be represented as due to imperfect elasticity. If the factor $1/Q$ for dissipation per cycle is assumed to be 1/100 for distortional strain energy and 1/1000 for dilatational strain energy, the heat now being generated in the moon by tides is less than 0.010 erg/g/yr.; i.e., negligible compared to radioactive heating by a chondritic composition. Tidal heating would be comparable to radioactive heating, however, if the semimajor axis of the moon's orbit was one-third as great, so the mechanism does limit the possible history of the moon's orbit. Also, appreciable tidal dissipation would cause thermal stresses and be conducive to convection, since it is much greater in the center than near the surface of the moon, and has a non-uniform distribution in latitude (and longitude as well, if rotation is synchronous with revolution).

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*Urey et al.* [1959] suggested that tidal dissipation may have significantly contributed to heating of the moon's interior in the past. *Kopal* [1963] has calculated the tidal dissipation in a moon assumed to be a fluid with Newtonian viscosity. However, since tidal distortion is a relatively high frequency phenomenon, it seems a more reasonable extrapolation from experience to assume that tidal dissipation in the moon occurs as a result of imperfect elasticity with a factor $1/Q$ for dissipation per cycle comparable to those estimated for the earth's mantle from polar tides, free oscillations, and latitude variation.

The tidal disturbing function $W$ is

$$W = \frac{GM^*}{r^*}\sum_{l=2}^{\infty}\left(\frac{r}{r^*}\right)^l P_l(\cos S) \qquad (1)$$

Where $P_l$ is a Legendre polynomial; $r, \phi, \lambda$ are radius, latitude, and longitude in a moon-fixed coordinate system; the asterisked quantities refer to the disturbing body; $GM^*$ is the product of the gravitational constant and the mass; and $S$ is the arc from $(\phi^*, \lambda^*)$ to $(\phi, \lambda)$. We apply the addition theorem to (1):

$$W = \frac{GM^*}{r^*}\sum_{l=2}^{\infty}\left(\frac{r}{r^*}\right)^l \sum_{m=0}^{l}\frac{(l-m)!}{(l+m)!}(2-\delta_{om})$$

$$\cdot P_{lm}(\sin\phi)P_{lm}(\sin\phi^*)$$

$$\cdot[\cos m\lambda \cos m\lambda^* + \sin m\lambda \sin m\lambda^*] \qquad (2)$$

Where $P_{lm}$ is the Legendre associated function and $\delta_{om}$ is the Kronecker delta.

To obtain the variation in time of $r^*, \phi^*, \lambda^*$, we must express them in terms of the earth's orbit referred to the moon. For the present orbit this is most simply done by using the numerical values of coefficients as given by Brown's theory of lunar motion, as recently described by *Harrison* [1963]. Because we wish to investigate the effects of changing the orbital parameters, and because the necessary computer subroutines had already been written for another purpose (to a degree of detail superfluous to the present problem), we used a different development [*Kaula*, 1961] based on the assumption that the orbit can be considered a Keplerian ellipse at a fixed inclination to the moon's equator with secularly moving node and perigee. For the tidal problem, the most significant omissions under this assumption are short-period perturbations of the semimajor axis and the longitude by the sun. For the present orbit, the largest of these terms (those containing $h$ in the arguments of equations 6 to 8 of *Harrison* [1963]) have a ratio of about 0.2 to the terms arising from the ellipticity of the orbit. This ratio would vary directly with variation in the semimajor axis of the lunar orbit but would stay about the same with variation in the inclination or eccentricity. It is consistent, then, with the unavoidable crudeness of our estimates of the dissipation factors $1/Q$ to assume a purely elliptic orbit.

We apply the transformation of equations 7 to



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28 in *Kaula* [1961] to $r^{*l-1}P_{lm}$ $(\sin \phi^*)$ $[\cos m\lambda^*, \sin m\lambda^*]$ in (2):

$$\begin{aligned}
W = {} & \frac{GM^*}{a^*} \sum_{l=2}^{\infty} \left(\frac{r}{a^*}\right)^l \\
& \cdot \sum_{m=0}^{l} \frac{(l-m)!}{(l+m)!} (2 - \delta_{0m}) P_{lm}(\sin \phi) \\
& \cdot \sum_{p,q} F_{lmp}(i^*) G_{lpq}(e^*) \\
& \cdot \left[ \cos m\lambda \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} \{(l-2p)\omega^* \right. \\
& + (l - 2p + q)M^* + m(\Omega^* - \theta)\} \\
& + \sin m\lambda \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} \{(l-2p)\omega^* \\
& \left. + (l - 2p + q)M^* + m(\Omega^* - \theta)\} \right]
\end{aligned} \tag{3}$$

where $a^*$, $e^*$, $i^*$, $\Omega^*$, $\omega^*$, and $M^*$ are the Keplerian elements of the earth's orbit referred to the moon's equator and a departure point thereon fixed with respect to inertial space; $F_{lmp}(i^*)$ and $G_{lpq}(e^*)$ are polynomials of the sine and cosine of the inclination and of the eccentricity, respectively, and $\theta$ is the 'lunar sidereal time': the angle between the inertially fixed departure point and the point on the moon from which selenographic longitudes are measured. We assume the rate $\dot{\theta}$ to be constant, which is equivalent to neglecting the physical libration.

We abbreviate (3) as

$$\begin{aligned}
W = {} & \sum_{l=2}^{\infty} r^l \sum_{m=0}^{l} P_{lm}(\sin \phi) \\
& \cdot \sum_{g} a_{lmg} \left[ \cos m\lambda \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \right. \\
& \left. + \sin m\lambda \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \right]
\end{aligned} \tag{4}$$

where the single subscript $g$ replaces the subscript pair $p$ and $q$; the amplitude

$$\begin{aligned}
a_{lmg} = {} & \frac{GM^*}{a^*} \left(\frac{r}{a^*}\right)^l \frac{(l-m)!}{(l+m)!} \\
& \cdot (2 - \delta_{om}) F_{lmp}(i^*) G_{lpq}(e^*)
\end{aligned} \tag{5}$$

and the rate

$$\begin{aligned}
\sigma_{lmg} = {} & (l - 2p)\dot{\omega}^* \\
& + (l - 2p + q)\dot{M}^* + m(\dot{\Omega}^* - \dot{\theta})
\end{aligned} \tag{6}$$

The strain energy per unit volume, divided into shear (or distortional) and compressive (or dilatational) terms [*Jeffreys*, 1959, p. 12] is

$$E = \mu e_{ij}' e_{ij}' + \frac{k}{6} e_{mm} e_{nn} \tag{7}$$

where summation is taken over repeated subscripts. In (7), $\mu$ is the rigidity, $k$ is the bulk modulus, $e_{ij}$ is the strain tensor and $e_{ij}'$ is the part of the strain tensor expressing departures from symmetry:

$$e_{ij}' = e_{ij} - \tfrac{1}{3}\delta_{ij} e_{kk} \tag{8}$$

where $\delta_{ij}$ is the Kronecker delta.

If the tidal disturbing function is expressed as a sum of spherical harmonics,

$$\begin{aligned}
W = {} & \sum_{l=2}^{\infty} r^l \sum_{m=0}^{l} P_{lm}(\sin \phi) \\
& \cdot [A_{lm}(t) \cos m\lambda + B_{lm}(t) \sin m\lambda]
\end{aligned} \tag{9}$$

the strain tensor $e_{ij}$ at any point $(r, \phi, \lambda)$ can be expressed as

$$\begin{aligned}
e_{ij} = {} & \sum_{l,m} [A_{lm}(t) \epsilon_{ijlmc}(r, \phi, \lambda) \\
& + B_{lm}(t) \epsilon_{ijlms}(r, \phi, \lambda)]
\end{aligned} \tag{10}$$

where $\epsilon_{ijlmc,s}$ is the response of a planetary model of specified shear modulus, bulk modulus, and density to a unit coefficient in the tidal disturbing function.

Comparing (4) and (9), we have

$$A_{lm}(t) = \sum_g a_{lmg} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \tag{11}$$

$$B_{lm}(t) = \sum_g a_{lmg} \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} (\sigma_{lmg} t - t_{lmg}) \tag{12}$$

Substituting (10), (11), and (12), into (7) and sorting out the algebra which results yields

$$\begin{aligned}
E = {} & \sum_{l,m,g,u,v,h} [(S_{1lmguvh} + C_{1lmguvh}) \\
& \cdot \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \{(\sigma_{lmg} + \sigma_{uvh})t - (t_{lmg} + t_{uvh})\} \\
& + (S_{2lmguvh} + C_{2lmguvh}) \\
& \cdot \begin{Bmatrix} \sin \\ \cos \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \{(\sigma_{lmg} + \sigma_{uvh})t - (t_{lmg} + t_{uvh})\}
\end{aligned}$$

$$+ (S_{3lmguvh} + C_{3lmguvh}) \cdot \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \{(\sigma_{lmg} - \sigma_{uvh})t - (t_{lmg} - t_{uvh})\}$$

$$+ (S_{4lmguvh} + C_{4lmguvh}) \cdot \begin{Bmatrix} -\sin \\ \cos \end{Bmatrix}_{b \text{ odd}}^{b \text{ even}} \{(\sigma_{lmg} - \sigma_{uvh})t - (t_{lmg} - t_{uvh})\} \Bigg] \tag{13}$$

where $b = l + u - m - v$; $(u, v)$ summations start at $(l, m)$; and

$$S_{1lmguvh} = a_{lmg}a_{uvh} \frac{\mu}{2} (\epsilon_{ijlmc}'\epsilon_{ijuvc}' - \epsilon_{ijlms}'\epsilon_{ijuvs}')(2 - \delta_{uv}^{lm})$$

$$C_{1lmguvh} = a_{lmg}a_{uvh} \frac{k}{12} (\epsilon_{iilmc}\epsilon_{jjuvc} - \epsilon_{iilms}\epsilon_{jjuvs})(2 - \delta_{uv}^{lm})$$

$$S_{2lmguvh} = a_{lmg}a_{uvh} \frac{\mu}{2} (\epsilon_{ijlmc}'\epsilon_{ijuvs}' + \epsilon_{ijlms}'\epsilon_{ijuvc}')(2 - \delta_{uv}^{lm}) \frac{(-1)^{u-v} + (-1)^{l-m}}{2}$$

$$C_{2lmguvh} = a_{lmg}a_{uvh} \frac{k}{12} (\epsilon_{iilmc}\epsilon_{jjuvs} + \epsilon_{iilms}\epsilon_{jjuvc})(2 - \delta_{uv}^{lm}) \frac{(-1)^{u-v} + (-1)^{l-m}}{2}$$

$$S_{3lmguvh} = a_{lmg}a_{uvh} \frac{\mu}{2} (\epsilon_{ijlmc}'\epsilon_{ijuvc}' + \epsilon_{ijlms}'\epsilon_{ijuvs}')(2 - \delta_{uv}^{lm})(-1)^{u-v}$$

$$C_{3lmguvh} = a_{lmg}a_{uvh} \frac{k}{12} (\epsilon_{iilmc}\epsilon_{jjuvc} + \epsilon_{iilms}\epsilon_{jjuvs})(2 - \delta_{uv}^{lm})(-1)^{u-v}$$

$$S_{4lmguvh} = a_{lmg}a_{uvh} \frac{\mu}{2} (\epsilon_{ijlmc}'\epsilon_{ijuvs}' - \epsilon_{ijlms}'\epsilon_{ijuvc}')(2 - \delta_{uv}^{lm}) \frac{1 - (-1)^b}{2}$$

$$C_{4lmguvh} = a_{lmg}a_{uvh} \frac{k}{12} (\epsilon_{iilmc}\epsilon_{jjuvs} - \epsilon_{iilms}\epsilon_{jjuvc})(2 - \delta_{uv}^{lm}) \frac{1 - (-1)^b}{2} \tag{14}$$

For any particular term of subscripts $l$, $m$, $u$, $v$, $g$, and $h$, the energy dissipated in one cycle of duration $2\pi/(\sigma_{lmg} + \sigma_{uvh})$ will be

$$\Delta E_{lmguvh+} = 2\pi[| S_{1lmguvh}/Q_s + C_{1lmguvh}/Q_c | + | S_{2lmguvh}/Q_s + C_{2lmguvh}/Q_c |] \tag{15}$$

and in one cycle of duration $2\pi/(\sigma_{lmg} - \sigma_{uvh})$

$$\Delta E_{lmguvh-} = 2\pi[| S_{3lmguvh}/Q_s + C_{3lmguvh}/Q_c | + | S_{4lmguvh}/Q_s + C_{4lmguvh}/Q_c |] \tag{16}$$

To obtain the contributions to energy dissipation per unit time, we multiply (15) and (16) by the absolute values of the rates per unit time, $|\sigma_{lmg} + \sigma_{uvh}|$ and $|\sigma_{lmg} - \sigma_{uvh}|$. We sum over all subscripts to obtain the mean dissipation rate at a particular point fixed in the moon:

$$\dot{E}_d(r, \phi, \lambda) = \sum_{l,m,g,u,v,h} [\Delta E_{lmguvh+} | \sigma_{lmg} + \sigma_{uvh} | + \Delta E_{lmguvh-} | \sigma_{lmg} - \sigma_{uvh} |] \tag{17}$$

If the rotation is synchronous with revolution, as it is at present,

$$\dot{\omega}^* + \dot{M}^* + \dot{\Omega}^* - \dot{\theta} = 0 \tag{18}$$

there are terms that will contribute only through their amplitude $a_{lmg}$ but not through their rate $\sigma_{lmg}$. In the synchronous case, a reference longitude must be fixed. If this reference longitude is the mean direction of the earth, all terms containing $\sin \{m(\omega^* + M^* + \Omega^* - \theta)\}$ are zero and all terms containing $\cos \{m(\omega^* + M^* + \Omega^* - \theta)\}$ are unity; i.e., in (10) the contribution to $A_{lm}(t)$ will be $a_{lmg}$ for $l - m$ even and 0 for $l - m$ odd, and to $B_{lm}(t)$ it will be 0 for $l - m$ even and $-a_{lmg}$ for $l - m$ odd. Practically the only term for which this effect is significant is $(l, m, p, q) = (2, 2, 0, 0)$.

Also, there will be degeneracies for $m = 0$ in all cases and for $m \neq 0$ in the synchronous case, requiring the combination of terms before proceeding as in equations 13 to 16. In these cases, the rate for the term of subscripts $(l, m, p, q)$ will be the negative of the rate for the term of subscript $(l, m, l - m - p, -q)$. If the rate is taken as that of the $(l, m, p, q)$ term, the amplitude for the cosine coefficient will be

$$a_{lmg} = a_{lmh} + (-1)^{l-m} a_{lmi} \tag{19}$$

and for the sine coefficient

$$b_{lmg} = a_{lmh} - (-1)^{l-m} a_{lmi} \tag{20}$$

where the subscript correspondence is $h$ with $(p, q)$ and $i$ with $(l - m - p, -q)$, and $a_{lmh}$ and $a_{lmi}$ are computed by (5). Then (14) must be modified so that $b_{lmg}$ and $b_{uvh}$ coefficients appear in front of the $\epsilon_{ijlmg}$, $\epsilon_{ijuvg}$, etc., terms in place of $a_{lmg}$ and $a_{uvh}$.

Another set of degeneracies occurring in the synchronous case arises because terms of subscript $(l, m, p, q)$ will have rates equal to terms of subscript $(l, m + 2i, p - i, q)$, where $i$ is any integer.

Including terms for which the disturbing function rate is zero in effect makes the energy dissipation rate a function of the constant value of the strain, which raises the question of whether strains from other than tidal causes should also be considered. Since we are interested in the dissipation over geological durations of time, these terms should perhaps be omitted because in such time we would expect nonoscillating strains to be removed by anelastic processes. But the dissipation rate obtained would then be an absolute minimum for the assumed $Q$. The moments of inertia of the moon indicate that it now contains strains larger than tidal, so leaving in the nonoscillating tidal terms should yield a dissipation rate unlikely to be too high.

The quantities $a_{lmg}$, $\sigma_{lmg}$, $l = 2$, and $m = 0$, 1, 2 were calculated from (5) and (6), for a variety of lunar orbits, using subroutines for $F_{lmp}(i^*)$ and $G_{lpq}(e^*)$ originally devised for analysis of close satellite orbits, and computing the rates $\dot{\omega}^*$, $\dot{M}^*$, and $\dot{\Omega}^*$ by the methods described by *Kaula* [1961].

The strain tensors $\epsilon_{ijlm}'$ and $\epsilon_{iilm}$ were calculated using the formulation of the earth-tide problem of *Alterman et al.* [1959], which has also been used by *Takeuchi et al.* [1962] and *Longman* [1963]. In this formulation, the basic variables are the radial factors of vector spherical harmonic expressions of the displacements, stresses, and potential terms: $y_1$, of the radial displacement; $y_2$, of the compressive stress; $y_3$, of the tangential displacement; $y_4$, of the shear stress; $y_5$, of the potential; and $y_6$, of the potential gradient less the contribution thereto of the radial displacement. The equations of equilibrium then become a system of six first-order equations:

$$\frac{dy_i}{dr} = P_{ij} y_j \tag{21}$$

The $P_{ij}$'s are functions of $r$, $k$, $\mu$, $g$, and $\rho$. Three columns of $P_{ij}$ have terms of $O(r^{-2})$, so that the requirement of regularity at the origin eliminates three constants of integration. The three surface conditions of zero tangential stress, zero radial stress, and the potential gradient being related to the potential as a spherical harmonic in free space make the problem determinate. After solving (21) numerically for the $y_i$'s corresponding to a particular harmonic $Y_{lm} = r^l S_{lm}$, the contribution to the strain matrix $\epsilon_{ij}$ is calculated by (here $\theta$ is colatitude, $\phi$ is longitude, and $\lambda$ is $k - \frac{2}{3}\mu$):

$$\begin{aligned}
\epsilon_{\theta\theta} &= 2\frac{S_{lm}}{r} y_1 + \frac{2}{r}\frac{\partial^2 S_{lm}}{\partial\theta^2} y_3 \\
\epsilon_{\phi\phi} &= 2\frac{S_{lm}}{r} y_1 + \frac{2}{r\sin\theta} \\
&\quad \cdot \left(\frac{1}{\sin\theta}\frac{\partial^2 S_{lm}}{\partial\phi^2} + \cos\theta\frac{\partial S_{lm}}{\partial\theta}\right) y_3 \\
\epsilon_{rr} &= -\frac{4\lambda}{\lambda + 2\mu}\cdot\frac{S_{lm}}{r} y_1 \\
&\quad + \frac{2l(l+1)\lambda}{\lambda + 2\mu}\cdot\frac{S_{lm}}{r} y_3 + \frac{2S_{lm}}{\lambda + 2\mu} y_2 \\
\epsilon_{\theta\phi} &= \frac{2}{r\sin\theta}\left(\frac{\partial^2 S_{lm}}{\partial\theta\partial\phi} - \cot\theta\frac{\partial S_{lm}}{\partial\phi}\right) y_2 \\
\epsilon_{\phi}r &= \frac{1}{\mu\sin\theta}\frac{\partial S_{lm}}{\partial\phi} y_4 \\
\epsilon r_{\theta} &= \frac{1}{\mu}\frac{\partial S_{lm}}{\partial\theta} y_4
\end{aligned} \tag{22}$$

Equation 22 can be derived using the equations of *Love* [1927, p. 56], applying a factor of ½ to the off diagonal components to be consistent

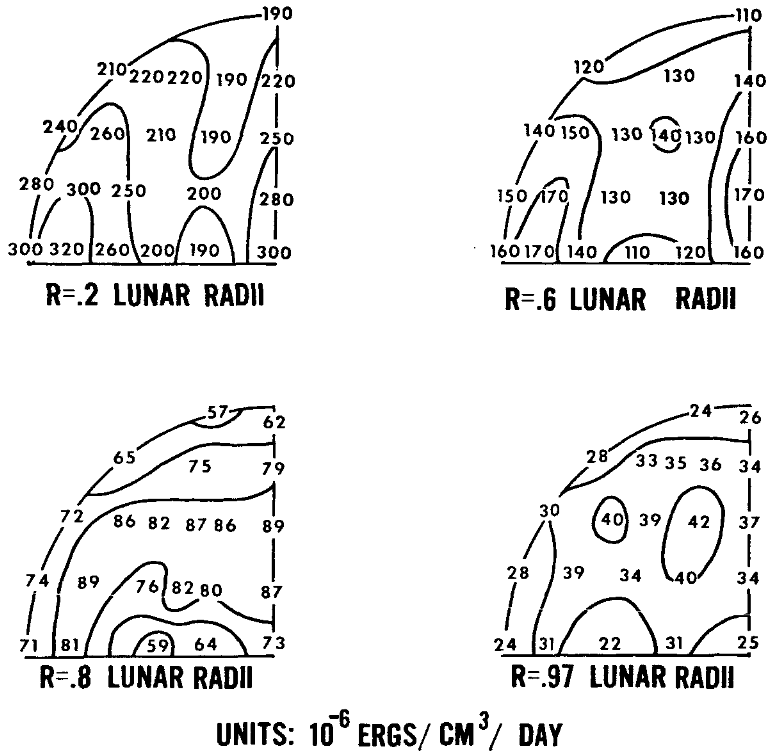


Fig. 1. Present tidal energy dissipation in the moon, assuming a shear $Q$ of 100 and a compressive $Q$ of 1000.

with tensor convention. The $\epsilon_{ijlm}$'s from the $Y_{lm}$'s were used in (14).

The pole tide suggests a $Q$ of more than 100 for the earth at a 14-month period [*Munk and MacDonald,* 1960, p. 162], while the latitude variations indicate a $Q$ of about 40 at the same period [*Munk and MacDonald,* 1960, p. 148; *Jeffreys,* 1959, pp. 255–259]. The free oscillations of the earth indicate $Q$'s of 170 to 400 for periods less than an hour [*Benioff et al.,* 1961; *Ness et al.,* 1961], except for a $Q$ of 7500 for the $S_0^0$, the only mode which is purely compressive [*Ness et al.,* 1961]. Models for rock creep proposed by *Jeffreys and Crampin* [1960], *J. R. MacDonald* [1961], and *Lomnitz* [1962] suggest $Q$'s between 40 and 100 for semimonthly and monthly periods. A shear $Q_s$ of 100 thus seems a reasonable compromise. Considering that excitation from the atmosphere and oceans may maintain the $S_0^0$ free oscillation [*Ness et al.,* 1961], we assume a compressive $Q_c$ of 1000.

The strain energies were calculated for several lunar models proposed by *Harrison* [1963]. However, since the uncertainty in $Q$ reduces this problem to one of estimating order of magnitude, this discussion will be limited to a homogeneous moon of density 3.34 g cm$^{-3}$, rigidity $7.38 \times 10^{11}$ dyne cm$^{-2}$, and bulk modulus $1.23 \times 10^{12}$ dyne cm$^{-2}$. The Love numbers obtained for this model by the numerical solution of (15) were 0.0344 for $h$, 0.0195 for $k$, and 0.0095 for $l$.

The thermal histories of lunar models with chondritic composition have been calculated by *G. J. F. MacDonald* [1959]. Even with a cold origin, these models come very close to melting at depths in the moon exceeding 500 km. The chondritic composition used had radioactive contents of $8.0 \times 10^{-4}$, $1.1 \times 10^{-8}$, and $4.4 \times 10^{-8}$ g/g for potassium, uranium, and thorium, respectively, which yield a thermal energy output of 1.59 ergs/g/year at present and 12.8 ergs/g/year $4.5 \times 10^9$ years ago. Hence for tidal dissipation to be significant, it should contribute of the order of 5 ergs/g/year, or 0.04 erg/cm$^3$/day.

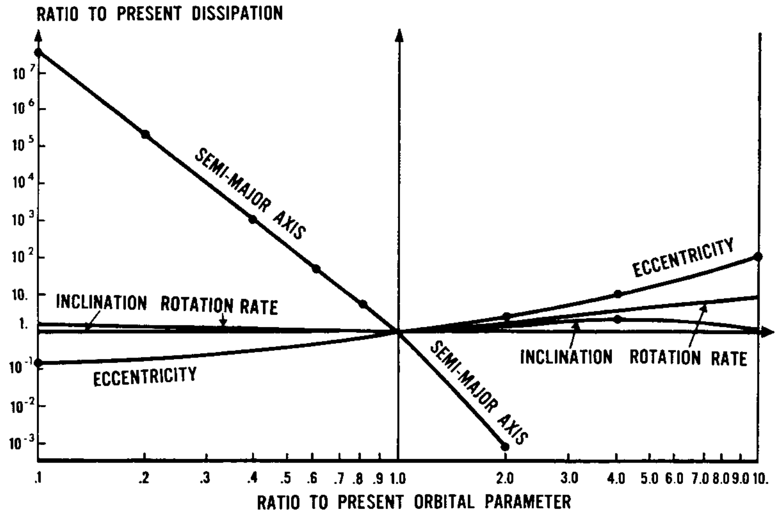


Fig. 2. Variation of tidal energy dissipation in the moon with one-at-a-time variation in orbital parameters.

The results for the present orbit of the moon are shown in Figure 1 in the form of maps of the energy dissipation at four levels within the moon. Since the dissipation is symmetric about the equator and two meridians at right angles, we need to show only one octant for each level. The evident features are, first, that the tidal dissipation is at present a negligible source of heat and, second, that the distribution of the heating is extremely nonuniform both radially and laterally. This variability of distribution suggests that, if the moon's orbit had ever been such that the over-all heating was appreciable, large thermal stresses would have resulted, leading to convection or some other form of mass motion.

The amplitudes and rates were therefore calculated for different orbital specifications. Each element was varied in turn, the others being held fixed at the present values: (1) the semimajor axis (holding the rotation synchronous with revolution about the earth); (2) the eccentricity; (3) the inclination; and (4) the rotation rate. The results are displayed in Figure 2, in the form of curves showing variation in the average ratio to the present dissipation with variation in the orbital elements. The variation is particularly marked with variation in the semimajor axis. If the semimajor axis were only one-third as great as it is now, the criterion of 0.04 erg/cm$^3$/day would be exceeded for most of the moon. At the secular acceleration calculated by *Munk and MacDonald* [1960], the moon would have been at this distance about $10^9$ years ago.

The pattern of energy dissipation shown in Figure 1 is composed of even-degree harmonics symmetric about the equator: (4, 4), (4, 2), (4, 0), (2, 2), and (2, 0). In a moon close enough so that heating by tidal dissipation was large enough to cause convection, the second-degree terms in its mass distribution would be determined mainly by the gravitational attraction of the earth. However, the fourth-degree terms would be determined by the convective pattern. It will therefore be interesting, when variations in the gravitational field and external form of the moon are better determined, to find out whether these fourth-degree terms are markedly larger than other terms, such as the third-degree terms. If they are, it will be a strong indication that the moon was once close enough for heating by tidal dissipation to cause convection, and hence it would lend further evidence as to the moon's origin.

In conclusion we can say that heating by tidal dissipation is currently insignificant (unless $Q$ factors estimated from the earth's mantle are wrong by a factor of about 100) but that this study confirms the suggestion of *Urey et al.* [1959] that it would have been important in the past if the moon were much closer to the earth.

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